Eur. Phys. J. A **16**, 209–219 (2003)
DOI 10.1140/epja/i2002-10084-x

THE EUROPEAN PHYSICAL JOURNAL A

Mixing and decays of ρ- and ω-mesons

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Received: 16 September 2002 /
Published online: 4 February 2003 –
Communicated by V.V. Anisovich

Abstract. Isospin-violating mixing of ρ- and ω-mesons is reconsidered in terms of propagators. Its influence on various pairs of (ρ^0, ω)-decays to the same final states is demonstrated. Some of them, $(\rho^0, \omega) \to \pi^+\pi^-$ and $(\rho^0, \omega) \to \pi^0\gamma$, have been earlier discussed in the literature, others (*e.g.*, $(\rho^0, \omega) \to \eta\gamma$ and $(\rho^0, \omega) \to e^+e^-$) are new in this context. Changes in partial widths for all the decay pairs are shown to be correlated. The set of present experimental data, though yet inconclusive, provides some limits for the direct $(\rho\omega)$-coupling and indirectly supports enhancement of $\rho^0 \to \pi^0\gamma$ in comparison with $\rho^\pm \to \pi^\pm\gamma$, though not so large as in some previous estimates.

PACS. 11.30.Ly Other internal and higher symmetries – 13.25.Jx Decays of other mesons – 14.40.Cs Other mesons with $S = C = 0$, mass < 2.5 GeV

1 Introduction

Isospin conservation was considered for many years to be a good symmetry of strong interactions, though violated due to electromagnetic (e.m.) corrections. Of course, e.m. mechanism of isospin violation should exist. However, the quark picture and QCD have suggested one more interesting possibility, to violate isospin by strong interactions as well. This is possible due to mass difference between u and d quarks. Parametrically, such mechanism could be stronger than the e.m. one, but its exact value essentially depends on unknown hadronic matrix elements and might appear numerically suppressed, at least, in some cases. In reality, for most of the known manifestations, the violation may look quantitatively compatible with the pure e.m. nature (numerically they are $\sim \mathcal{O}(\alpha)$ in amplitudes, or even smaller). Therefore, very elaborate work, both theoretical and experimental, will be necessary to pick out the underlying physics and separate various sources of isospin violation.

A special situation appears in the decay $\omega \to \pi^+\pi^-$ (branching ratio about 2% [1]), where enhancement becomes possible (and seems to be operative) due to transition of ω into ρ^0 having the near mass and large width $\rho^0 \to \pi^+\pi^-$. However, experiments with this mode can extract only one real parameter (instead of two or more, see below for details) and, thus, are insufficient to reveal the isospin violation mechanism(s).

In this respect, one more decay mode has attracted much attention in recent years. It is the radiative decay $\rho^0 \to \pi^0\gamma$. Its partial width was expected to be the same as for the charged companion $\rho^\pm \to \pi^\pm\gamma$. Meanwhile, experiment seems to give evidence [1] for a higher branching ratio of the neutral mode as compared to the charged one, though the result might still change[1]. Qualitatively, it may have reasonable explanation as being due to mixture of the direct decay and the cascade transition through ω with the relatively large amplitude $\omega \to \pi^0\gamma$.

A quantitative consideration has been mainly done in terms of a kind of effective field theory with some model Lagrangian (like Vector Dominance Model, Chiral Perturbation Theory and so on, see ref. [2] and many references therein). Such approaches, to be applicable, need some limitations (*e.g.*, constant and real transition vertices), which may appear too restrictive. Another approach, in terms of propagators, was applied more recently [3]. Motivated by summing general Feynman graphs, it gave an unexpectedly large enhancement for $\rho^0 \to \pi^0\gamma$.

In the present paper this approach is reconsidered more accurately. The consideration is then extended further to show that the $(\rho\omega)$-mixing should affect a wider set of decay modes where effects of mixing should be possible as well providing enhancements or suppressions of partial widths. Indeed, the present data qualitatively confirm the expected role of mixing. Such a way, at better experimental accuracy, may help to construct a consistent picture of the isospin violation and to clarify its nature.

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[1] One should make some reservations here. Measurement methods are very different: Primakoff effect for $\rho^\pm$, and e^+e^--annihilation for ρ^0. Backgrounds are also very different and not quite clear for ρ^0, see short discussion below.

What about the enhancement suggested in ref. [3], it is shown to be strongly overestimated.

2 Propagator description for mixing of vector particles

The unperturbed propagator for a vector meson V with "bare" mass $M_V^{(0)}$ may be presented in the form

$$[D_V^{(0)}(k^2)]_{\mu\nu} = \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_V^{(0)2}}}{k^2 - M_V^{(0)2}} = \frac{1}{k^2 - M_V^{(0)2}} \times \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - \frac{1}{M_V^{(0)2}} \frac{k_\mu k_\nu}{k^2}\,. \qquad (1)$$

Note that $(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$ and $\frac{k_\mu k_\nu}{k^2}$ provide a complete set of projectors since

$$\frac{k_\mu k_\alpha}{k^2}\, g^{\alpha\beta}\, \frac{k_\beta k_\nu}{k^2} = \frac{k_\mu k_\nu}{k^2}\,,$$
$$\left(g_{\mu\alpha} - \frac{k_\mu k_\alpha}{k^2}\right) g^{\alpha\beta} \left(g_{\beta\nu} - \frac{k_\beta k_\nu}{k^2}\right) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right),$$
$$\left(g_{\mu\alpha} - \frac{k_\mu k_\alpha}{k^2}\right) g^{\alpha\beta}\, \frac{k_\beta k_\nu}{k^2} = 0\,,$$
$$\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{k_\mu k_\nu}{k^2} = g_{\mu\nu}\,. \qquad (2)$$

The most general form of the vertex for the two-vector-mesons transition $V_1 \to V_2$ also contains two terms, transversal and longitudinal:

$$[\Gamma_{12}(k^2)]_{\mu\nu} = G_{12}(k^2) \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + F_{12}(k^2)\, \frac{k_\mu k_\nu}{k^2}\,. \qquad (3)$$

The vertex for the transition $V_2 \to V_1$ is similar. Moreover, T-invariance makes it just the same. We retain, however, formal difference of, say, G_{12} and G_{21} to reveal the structure of arising expressions.

Now we can describe the evolution of any initial state. The full propagator for $V_1 \to V_1$ is

$$D_{11} = D_1^{(0)} + D_1^{(0)} \Gamma_{12} D_2^{(0)} \Gamma_{21} D_1^{(0)} + D_1^{(0)} \Gamma_{12} D_2^{(0)} \Gamma_{21} D_1^{(0)} \Gamma_{12} D_2^{(0)} \Gamma_{21} D_1^{(0)} + \dots\,.$$

The summation runs separately for each of the projector terms due to their orthogonality, so we obtain

$$[D_{11}]_{\mu\nu} = (k^2 - M_2^{(0)2})\, R_t(k^2) \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - M_2^{(0)2}\, R_l(k^2)\, \frac{k_\mu k_\nu}{k^2} \qquad (4)$$

with

$$R_t(k^2) = [(k^2 - M_1^{(0)2})(k^2 - M_2^{(0)2}) - G_{12}G_{21}]^{-1}\,,$$
$$R_l(k^2) = [M_1^{(0)2} M_2^{(0)\,2} - F_{12}F_{21}]^{-1}\,. \qquad (5)$$

The full propagator for the transition $V_1 \to V_2$ takes the form

$$[D_{12}]_{\mu\nu} = G_{12}\, R_t(k^2) \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + F_{12}\, R_l(k^2)\, \frac{k_\mu k_\nu}{k^2}\,. \qquad (6)$$

Full propagators for transitions $V_2 \to V_2$ and $V_2 \to V_1$ can be obtained from eqs. (4), (6) by interchange of the indices 1 and 2. In all the expressions one may, generally, consider $M_1^{(0)}$, $M_2^{(0)}$ to be also k^2-dependent. The above description can be applied to mixing of any vector mesons (*e.g.*, φ and ω). It may even be generalized to mixing of any number of mesons (say, ρ-ω-φ, or admixture of radially excited states).

For the particular case of (ρ,ω)-mixing the formulae simplify. It is, first of all, due to nearness of the "bare" masses: $\left|\delta M^2/M^2\right| \sim 0.1$, where

$$\delta M^2 = \frac{M_\omega^{(0)2} - M_\rho^{(0)2}}{2}\,, \quad M^2 = \frac{M_\omega^{(0)2} + M_\rho^{(0)2}}{2}\,, \qquad (7)$$

$$M_{\omega,\rho}^{(0)} = m_{\omega,\rho}^{(0)} - \frac{i}{2}\, \Gamma_{\omega,\rho}^{(0)} \qquad (8)$$

(we take masses and widths from Particle Data Tables [1]). As a result, the essential k^2-region is small, in the vicinity of the masses, and that allows one to consider the transition vertices as constants: say, $G(k^2) \to G(M^2)$. The constancy of vertices corresponds to what is assumed in effective field theories. However, the effective vertex G may appear complex, while it should be real for self-consistency of field theory. (More exactly, in the field theory one should be able to change the phase of G by rephasing the fields ω and ρ; in this way one can make $\arg G = 0$. If, however, G contains contributions of real intermediate states, such as 2π and 3π, then the $\omega\rho$-rephasing may be insufficient to assure real G.) Corrections for k^2-dependence, when taken in the framework of an effective field theory, may also provide difficulties.

At constant vertices, the longitudinal part R_l has no poles in k^2 (and no k^2-dependence at all), while R_t has two poles corresponding to "physical" states ρ and ω (cf. with the structure of the unmixed propagator (1)). The "physical" masses are equal:

$$M_\omega^2 = M^2 + K\delta M^2\,, \qquad M_\rho^2 = M^2 - K\delta M^2\,, \qquad (9)$$

where

$$K = \sqrt{1 + \widetilde{G}_{\rho\omega}\widetilde{G}_{\omega\rho}}\,, \quad \widetilde{G}_{\rho\omega} = \frac{G_{\rho\omega}}{\delta M^2}\,, \quad \widetilde{G}_{\omega\rho} = \frac{G_{\omega\rho}}{\delta M^2}\,. \qquad (10)$$

Let us consider a process $i \to f$ with intermediate ρ- and ω-mesons. Its amplitude is

$$A_{if} = A_{i\rho}^{(0)} D_{\rho\rho} A_{\rho f}^{(0)} + A_{i\rho}^{(0)} D_{\rho\omega} A_{\omega f}^{(0)} + A_{i\omega}^{(0)} D_{\omega\omega} A_{\omega f}^{(0)} + A_{i\omega}^{(0)} D_{\omega\rho} A_{\rho f}^{(0)}\,, \qquad (11)$$

where $A_{i\rho}^{(0)}, A_{i\omega}^{(0)}$ are production amplitudes for "bare" ρ-, ω-states, while $A_{\rho f}^{(0)}, A_{\omega f}^{(0)}$ are their decay amplitudes. We will be really interested here in decay modes (e^+e^-, $\pi^0\gamma$ and some others) with current conservation, thus only the transversal parts of the propagators are operative. Then we can rewrite the amplitude through contributions of the "physical" states

$$A_{if} = A_{i\rho} D_\rho A_{\rho f} \, + \, A_{i\omega} D_\omega A_{\omega f} \, , \tag{12}$$

with "physical" propagators

$$[D_\rho(k^2)]_{\mu\nu} = \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_\rho^2}}{k^2 - M_\rho^2} \, , \quad [D_\omega(k^2)]_{\mu\nu} = \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_\omega^2}}{k^2 - M_\omega^2} \tag{13}$$

(see eq. (9) for masses) and "physical" amplitudes

$$A_{i\rho} = \sqrt{\frac{K+1}{2K}} \left(A_{i\rho}^{(0)} - A_{i\omega}^{(0)} \frac{\widetilde{G}_{\omega\rho}}{K+1} \right) ,$$
$$A_{i\omega} = \sqrt{\frac{K+1}{2K}} \left(A_{i\omega}^{(0)} + A_{i\rho}^{(0)} \frac{\widetilde{G}_{\rho\omega}}{K+1} \right) , \tag{14}$$

for the meson production and

$$A_{\rho f} = \sqrt{\frac{K+1}{2K}} \left(A_{\rho f}^{(0)} - \frac{\widetilde{G}_{\rho\omega}}{K+1} A_{\omega f}^{(0)} \right) ,$$
$$A_{\omega f} = \sqrt{\frac{K+1}{2K}} \left(A_{\omega f}^{(0)} + \frac{\widetilde{G}_{\omega\rho}}{K+1} A_{\rho f}^{(0)} \right) , \tag{15}$$

for meson decays. The structure of masses (9) and amplitudes (14), (15) corresponds to diagonalizing the mass squared matrix of the (ρ, ω)-system,

$$\mathcal{M}^2 = \begin{pmatrix} M_\rho^{(0)\,2} & G_{\omega\rho} \\ G_{\rho\omega} & M_\omega^{(0)\,2} \end{pmatrix} , \tag{16}$$

and its matrix propagator, $\mathcal{D} = (k^2 - \mathcal{M}^2)^{-1}$, in the form

$$\mathcal{M}^2 = \sqrt{\frac{K+1}{2K}} \begin{pmatrix} 1 & \frac{\widetilde{G}_{\omega\rho}}{K+1} \\ -\frac{\widetilde{G}_{\rho\omega}}{K+1} & 1 \end{pmatrix} \cdot \begin{pmatrix} M_\rho^2 & 0 \\ 0 & M_\omega^2 \end{pmatrix} \cdot \sqrt{\frac{K+1}{2K}} \begin{pmatrix} 1 & -\frac{\widetilde{G}_{\omega\rho}}{K+1} \\ \frac{\widetilde{G}_{\rho\omega}}{K+1} & 1 \end{pmatrix} . \tag{17}$$

Additional simplifications arise from T-invariance which allows one to choose phases of states, so that $G_{\rho\omega} = G_{\omega\rho}$. Further, from previous experience of isospin violation we expect the $(\rho\omega)$-transition vertices to be numerically small. *E.g.*, the e.m. mechanism gives $|G|, |F| \sim \mathcal{O}(\alpha) \cdot M^2 \sim 10^{-2}\, M^2$. Then $|\widetilde{G}_{\rho\omega}| \sim 0.1$. In such a case $|K-1| \sim 10^{-2}$, and with sufficient accuracy we can substitute $K = 1$ in eqs. (14), (15), (17). Corrections for deviation of K from unity correspond to accounting for cascade returns $\rho \to \omega \to \rho$ and/or $\omega \to \rho \to \omega$.

The above picture of $(\rho\omega)$-mixing is quite similar to the well-known mixing of $(K^0\overline{K}^0)$ as described by Lee, Oehme, Yang [4]. The bare states $|\rho^{(0)}\rangle$ and $|\omega^{(0)}\rangle$ appear to be analogs of $|K^0\rangle$ and $|\overline{K}^0\rangle$, while the physical states

$$|\rho\rangle = N_\rho \left(|\rho^{(0)}\rangle - \frac{\widetilde{G}_{\rho\omega}}{K+1} |\omega^{(0)}\rangle \right) ,$$
$$|\omega\rangle = N_\omega \left(\frac{\widetilde{G}_{\omega\rho}}{K+1} |\rho^{(0)}\rangle + |\omega^{(0)}\rangle \right) \tag{18}$$

play the role of $|K_S\rangle$ and $|K_L\rangle$ (compare with expressions (15); N_ρ and N_ω are normalizing factors). The essential difference, however, is $\delta M^2 \neq 0$, which would imply CPT violation in the case of $(K^0\overline{K}^0)$.

This similarity reveals one more property of the $(\rho\omega)$-system. In the case of exact isospin conservation the bare states $\rho^{(0)}$ and $\omega^{(0)}$ cannot be coherent, and their phases (absolute and/or relative) are totally independent. If mixing is possible, the physical states ρ and ω can be coherent to each other, so their relative phase becomes physically meaningful. Nevertheless, phases of the bare states stay arbitrary and may be changed so to not change phases of the physical states (of course, phases of the normalizing factors and of $\widetilde{G}_{\omega\rho}$, $\widetilde{G}_{\rho\omega}$ should be changed correspondingly). Such procedure, rephasing, is familiar in the description of neutral kaons, with only rephasing-invariant quantities being physically meaningful and measurable. It may be useful also for the $(\rho\omega)$-system.

For the $(K^0\overline{K}^0)$-system with CPT conservation we know that the states $|K_S\rangle$ and $|K_L\rangle$ would be mutually orthogonal only if T (or CP) were conserved. The bare states $|\rho^{(0)}\rangle$ and $|\omega^{(0)}\rangle$ in the $(\rho\omega)$-system are, surely, orthogonal to each other, but the physical states $|\rho\rangle$ and $|\omega\rangle$ can be also orthogonal only if

$$\frac{\widetilde{G}_{\rho\omega}}{K+1} = \frac{\widetilde{G}^*_{\omega\rho}}{K^*+1} \, . \tag{19}$$

Evidently, this condition implies $|\widetilde{G}_{\rho\omega}| = |\widetilde{G}_{\omega\rho}|$ (*i.e.*, $|G_{\rho\omega}| = |G_{\omega\rho}|$), which would be provided by T-invariance. Hence, the T-invariance is necessary for (ρ,ω)-orthogonality, in similarity with neutral kaons. It is not, however, sufficient. Equation (19) is consistent with definition (10) only if K (and $\widetilde{G}_{\rho\omega}\widetilde{G}_{\omega\rho}$) is real. Note that combinations $\widetilde{G}_{\rho\omega}\widetilde{G}_{\omega\rho}$ (and K as well) and $\widetilde{G}_{\rho\omega}/\widetilde{G}^*_{\omega\rho}$ are rephasing invariant, *i.e.*, not changed if phases of $|\rho^{(0)}\rangle$ and $|\omega^{(0)}\rangle$ change under fixed phases of $|\rho\rangle$ and $|\omega\rangle$. Thus, the necessary and sufficient condition of the (ρ,ω)-orthogonality is the possibility to choose such phases of the bare states that $\widetilde{G}_{\rho\omega}$ and $\widetilde{G}_{\omega\rho}$ are equal and real. This condition may appear not natural (see discussion below and recall that the $\widetilde{G}$'s contain the complex denominator δM^2), so, most probably, the mixed physical eigenstates are non-orthogonal even in spite of the T conservation.

There is one more similarity between the $(\rho\omega)$ and $(K^0\overline{K}^0)$ systems. CP violation for neutral kaons can manifest itself in two forms: the mixing violation due to a CP-violating structure of the kaon effective Hamiltonian, and the direct violation related directly to kaon decay amplitudes. Isospin violation in the $(\rho\omega)$-system may, analogously, have two forms: the mixing violation due to the isospin-violating structure of the mass squared matrix (16) (nonvanishing $G_{\rho\omega}$ and/or $G_{\omega\rho}$), and the direct violation (nonvanishing, even if being isospin-forbidden, amplitudes $A_\rho^{(0)}, A_\omega^{(0)}$ for the bare meson production and/or decay amplitudes).

If we could produce pure states $\rho^{(0)}$, $\omega^{(0)}$ and observe their decays in real time, we would see oscillating time distributions (analogues of oscillating decays of pure K^0 and $\overline{K}^0$). But this is surely unrealistic because of too short lifetimes, and experiments can study only the two-pole structure of time-integrated k^2-dependencies. It should be emphasized that in any experiment one can extract only those poles related to physical ρ-, ω-states, with residues containing physical amplitudes (14), (15). The bare states $\rho^{(0)}$, $\omega^{(0)}$ and their amplitudes are unobservable.

The latter discussions in terms of states has implicitly assumed that bare amplitudes, bare masses and vertices $G_{\rho\omega}$, $G_{\omega\rho}$ are constants. However, such assumptions are not necessary. All expressions (7)-(17) conserve their form if (all or some of) the above quantities depend on k^2. Then the mixing parameters $\widetilde{G}_{\rho\omega}$, $\widetilde{G}_{\omega\rho}$, K, as well as the "physical amplitudes" and "physical masses" (of course not pole ones) become also k^2-dependent.

3 $(\rho\,\omega)$-mixing in particular decay modes

The most popular in the literature on $(\rho\,\omega)$ isospin violation is the meson mixing described by the parameters $\widetilde{G}$, since they reveal some enhancement due to the small value of δM^2 in denominator, see definition (10). With the reasonable assumption of T-invariance we can choose phases of the bare states so to have one (generally, complex) dimensionless parameter $\widetilde{G}_{\rho\omega} = \widetilde{G}_{\omega\rho}$, which is universal in all particular processes. In an effective field theory the related parameters $G_{\rho\omega}, G_{\omega\rho}$ appear in the Lagrangian as coupling constants for direct transitions $\omega \rightleftharpoons \rho$. Due to Hermiticity, they may be taken equal and real (the corresponding terms by themselves are inevitably T-invariant). Even in this case the complexity of $\widetilde{G}_{\rho\omega} = \widetilde{G}_{\omega\rho}$ cannot be removed; it is totally determined by the complexity of δM^2, due to the widths of ρ and ω. Note that for the current experimental values of masses and widths [1]

$$2\delta M^2 = M_\omega^2 - M_\rho^2 = (23368 + 108443i)\,\mathrm{MeV}^2\,,$$

i.e., δM^2 is mainly imaginary, due to the large Γ_ρ.

The realistic situation may be different. Transitions $\omega \rightleftharpoons \rho$ may go through some intermediate states, virtual or real. If only virtual states are possible (say, for the transition $\omega \to K\bar{K} \to \rho$ advocated in [2]), then $G_{\rho\omega} = G_{\omega\rho}$ are pure real indeed. However, if real intermediate states (say, pions or pions with one photon) are also essential, then $G_{\rho\omega} = G_{\omega\rho}$ should be complex by themselves. Correspondingly, parameters $\widetilde{G}_{\rho\omega} = \widetilde{G}_{\omega\rho}$, being also universal, should have additional complexity, not related to δM^2. Such a case is surely out of the framework of an effective field theory.

Apart from mixing, the isospin violation can manifest itself directly in amplitudes of production and/or decay of bare states $\rho^{(0)}$ and $\omega^{(0)}$ (see eqs. (14), (15)). Intuitively, such contributions have no enhancement and should be smaller than the enhanced mixing violation of isospin. However, this may appear not universally true. In particular, effective mechanisms for direct and mixing violations may appear different. This could make the direct isospin violation be essential in some processes, though negligible in others (again, phenomenologically similar to apparent properties of CP violation).

In any case, at the present state of knowledge and experience one needs to use some model assumptions on the amplitudes and mixing. That is why separate considerations are applied in the present paper to particular ρ- and/or ω-decays.

3.1 Decays $(\omega, \rho) \to \pi^+\pi^-$

The final state $\pi^+\pi^-$ in these decays has isospin $I = 1$. Hence, the standard (and reasonable) assumption is that the direct amplitude for $\omega^{(0)} \to \pi^+\pi^-$ vanishes (or is very small), and the decay goes only, or at least mainly, through mixing[2]. Then eq. (15) with $|\widetilde{G}_{\omega\rho}| \ll 1$ leads to

$$A(\omega \to \pi^+\pi^-) = \frac{\widetilde{G}_{\omega\rho}}{2}\,A(\rho \to \pi^+\pi^-)\,,$$
$$\Gamma(\omega \to \pi^+\pi^-) = \frac{|\widetilde{G}_{\omega\rho}|^2}{4}\,\Gamma_\rho\,. \qquad (20)$$

The present experimental data [1] lead to

$$\Gamma(\omega \to \pi^+\pi^-) = (144\pm24)\,\mathrm{keV}\,, \quad \Gamma_\rho = (149.2\pm0.7)\,\mathrm{MeV}$$

and provide

$$|\widetilde{G}_{\omega\rho}| = (6.2 \pm 0.5)\cdot 10^{-2}\,, \qquad (21)$$

in good agreement with qualitative expectations (see the brief discussion after eq. (17)). Evidently, the phase of $\widetilde{G}_{\omega\rho}$ cannot be determined by using only this pair of decays.

Together with data [1] on masses and total widths for ρ- and ω-mesons we obtain

$$|G_{\omega\rho}| = |\widetilde{G}_{\omega\rho}\,\delta M^2| = (3.44 \pm 0.29)\,10^{-3}\ \mathrm{GeV}^2\,, \qquad (22)$$

in agreement with phenomenological estimates of other authors and even with some theoretical estimates.

The small values (21) for $|\widetilde{G}_{\omega\rho}|$ and (22) for $|G_{\omega\rho}|$ cannot, by themselves, discriminate between different mechanisms of isospin violation (say, electromagnetic, or related to a definite hadronic channel, or any other). Phases of those parameters, being extracted from experimental data, would be very helpful.

[2] There are, however, theoretical estimates with not very small direct $\omega\pi\pi$-transition, see, *e.g.* [5].

One more note is reasonable here. The error for $|\widetilde{G}_{\omega\rho}|$ in eq. (21) looks rather small ($< 10\%$). However, the true uncertainty seems to be higher. *E.g.*, parameters given in the previous Particle Data Table [6] lead to

$$|\widetilde{G}_{\omega\rho}| = (7.0 \pm 0.5) \cdot 10^{-2} ,$$

with deviation about 2σ from the value (21). That is why we will use 2σ level as the uncertainty of $|\widetilde{G}_{\omega\rho}|$ in various numerical estimates below.

3.2 Decays $(\omega, \rho) \to \pi\gamma$

Isospin conservation does not forbid the direct transitions, both $\rho^{(0)} \to \pi^0\gamma$ and $\omega^{(0)} \to \pi^0\gamma$, since γ-quantum has two isospin components. Therefore, we need some information on the relation between the two amplitudes. The corresponding exact predictions are still absent, and some models should be used. Here we will apply the relations

$$A^{(0)}(\omega \to \pi^0\gamma) = 3\,A^{(0)}(\rho^0 \to \pi^0\gamma) = 3\,A^{(0)}(\rho^\pm \to \pi^\pm\gamma) , \qquad (23)$$

that were predicted years ago [7–10] on the basis of the quark model (in the form known at present as the additive quark model). They were derived with two main assumptions: 1) mesons consist of one quark-antiquark pair, 2) quark charges have their conventional values. In particular, the coefficient 3 is really a combination of charges:

$$3 = \frac{e_u - e_d}{e_u + e_d} .$$

Note also that eq. (23) needs a special choice of the relative phase between $\omega^{(0)}$ and $\rho^{(0)}$. As a matter of fact, the phase was fixed by standard assumptions that $\omega^{(0)} = (u\overline{u} + d\overline{d})/\sqrt{2}$, $\rho^{(0)} = (u\overline{u} - d\overline{d})/\sqrt{2}$.

More refined approaches, like QCD sum rules, lead to nearly the same relations, but with much more complicated derivations, which become sometimes a kind of art. The limit $N_c \to \infty$, suggested in ref. [3] as a basis for relation (23), is not adequate. It does provide mesons containing only one quark-antiquark pair, but quark charges should be N_c-dependent to prevent the triangle anomaly in the Standard Model. Hence, this limit would give different coefficients for eq. (23). (For a more detailed discussion of difficulties of the Standard Model at $N_c \to \infty$ see ref. [11].)

The $(\rho\omega)$-mixing does not affect the decay $\rho^\pm \to \pi^\pm\gamma$, and we can compare it with other decays to check predictions of the mixing picture.

If relations (23) are correct, the physical amplitude for $\omega \to \pi^0\gamma$ is practically the same as $A^{(0)}(\omega \to \pi^0\gamma)$, and the ratio of widths for $\omega \to \pi^0\gamma$ and $\rho^\pm \to \pi^\pm\gamma$

$$r_{\omega/\rho\pm\pi} \equiv \frac{\Gamma(\omega \to \pi^0\gamma)}{\Gamma(\rho^\pm \to \pi^\pm\gamma)} = 9\left|1 + \frac{1}{6}\,\widetilde{G}_{\omega\rho}\right|^2 \qquad (24)$$

is nearly independent of the mixing.

The present experimental data [1] give

$$\Gamma(\omega \to \pi^0\gamma) = (734 \pm 34)\ \text{keV} ,$$
$$\Gamma(\rho^\pm \to \pi^\pm\gamma) = (67.1 \pm 7.5)\ \text{keV} ,$$
$$r_{\omega/\rho\pm\pi} = (10.9 \pm 1.3) . \qquad (25)$$

This value reasonably agrees with the "bare" expectation of 9. If the deviation from 9 is, nevertheless, definitely confirmed, it could be a result of $(\omega\rho)$-mixing. However, such possibility looks doubtful, since the mixing correction in eq. (24) with the value (21) cannot exceed 3%. Furthermore, the mixing interpretation of the value (25) requires $\mathrm{Re}\,\widetilde{G}_{\omega\rho} > 0$, while other decay data, more sensitive to mixing, prefer $\mathrm{Re}\,\widetilde{G}_{\omega\rho} < 0$ (see below). Therefore, more reasonable would be to admit deviation of the coefficient in eq. (23) from 3. Taking literally, the value (25) without mixing leads to 3.3 instead of 3. Note that the increase of this coefficient would diminish the coefficient before $\widetilde{G}_{\omega\rho}$ in eq. (24), 1/6.6 instead of 1/6, and, hence, would diminish the mixing influence on the decay $\omega \to \pi^0\gamma$.

Neutral decay mode $\rho^0 \to \pi^0\gamma$ should be stronger affected by mixing. Combining eqs. (15), (23), we obtain its relative width in respect to $\rho^\pm \to \pi^\pm\gamma$ in the form

$$r_{\rho 0/\rho\pm\pi} \equiv \frac{\Gamma(\rho^0 \to \pi^0\gamma)}{\Gamma(\rho^\pm \to \pi^\pm\gamma)} = \left|1 - \frac{3}{2}\,\widetilde{G}_{\rho\omega}\right|^2 . \qquad (26)$$

Now we can apply T-invariance and use the value (21). Nevertheless, because of the unknown phase of the mixing parameter one can determine only boundaries for $r_{\rho 0/\rho\pm\pi}$, but not its value. With possible 2σ deviation for $|\widetilde{G}_{\rho\omega}|$ we obtain

$$0.80 \le r_{\rho 0/\rho\pm\pi} \le 1.23 . \qquad (27)$$

Then the current value $\mathrm{Br}\,(\rho^\pm \to \pi^\pm\gamma) = (4.5 \pm 0.5) \cdot 10^{-4}$ [1] gives

$$3.2 \cdot 10^{-4} \le \mathrm{Br}\,(\rho^0 \to \pi^0\gamma) \le 6.1 \cdot 10^{-4} . \qquad (28)$$

Now, if we knew the reliable experimental value of $r_{\rho 0/\rho\pm\pi}$, we would be able to separate $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$ and $|\mathrm{Im}\,\widetilde{G}_{\rho\omega}|$ in addition to the value (21) for $|\widetilde{G}_{\rho\omega}|$. Note that higher/lower values of $r_{\rho 0/\rho\pm\pi}$ correspond to negative/positive values of $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$. Thus, enhancement/suppression of $\rho^0 \to \pi^0\gamma$ in respect to $\rho^\pm \to \pi^\pm\gamma$ implies negative/positive sign of $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$.

It is worth to emphasize that such correlation is totally independent of the exact value of the coefficient in eq. (23). Therefore, even not too accurate experimental comparison of neutral and charged modes of $\rho \to \pi\gamma$ directly and reliably measures the sign of $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$.

If one neglects the mixing influence on $\mathrm{Br}\,(\omega \to \pi^0\gamma)$ and uses the empirical value (25) to correct the coefficient in eq. (23), then the coefficient before $\widetilde{G}_{\rho\omega}$ in eq. (26) increases, 3.3/2 instead of 3/2, thus increasing the mixing influence on $\rho^0 \to \pi^0\gamma$. Numerically, however, boundaries of intervals (27), (28) stay nearly the same.

We can also construct one more ratio

$$r_{\omega/\rho 0\pi} \equiv \frac{\Gamma(\omega \to \pi^0\gamma)}{\Gamma(\rho^0 \to \pi^0\gamma)} = 9 \left| \frac{1 + \frac{1}{6}\,\widetilde{G}_{\omega\rho}}{1 - \frac{3}{2}\,\widetilde{G}_{\rho\omega}} \right|^2 \qquad (29)$$

with boundaries

$$7.2 \leq r_{\omega/\rho 0\pi} \leq 11.6\,. \qquad (30)$$

Note that the lower/upper boundary in eq. (30) corresponds to the upper/lower boundaries for intervals (27), (28) and to the negative/positive sign of $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$.

Theoretical estimations for $\rho^0 \to \pi^0\gamma$, as a rule, agree with the phenomenological intervals (27), (28), with tendency to their upper ends (see, *e.g.*, ref. [2]). The only exclusion is the essentially higher estimate [3]. It is interesting to trace the source of such deviation. Detailed comparison shows that instead of

$$-\frac{1}{2}\,\widetilde{G}_{\rho\omega} = \frac{G_{\rho\omega}}{M_\rho^2 - M_\omega^2}$$

the amplitude of ref. [3] contains the quantity

$$-\frac{1}{2}\,\widetilde{G}'_{\rho\omega} = \frac{G_{\rho\omega}}{m_\rho^2 - m_\omega^2 + i\,m_\omega \Gamma_\omega}$$

with the same value of $G_{\rho\omega}$ (up to uncertainties and notations). At the current values of masses and widths [1] this provides the additional enhancing factor

$$\left| \frac{M_\rho^2 - M_\omega^2}{m_\rho^2 - m_\omega^2 + i\,m_\omega\Gamma_\omega} \right| = \left| \frac{m_\rho^2 - m_\omega^2 - i\,m_\rho\Gamma_\rho + i\,m_\omega\Gamma_\omega}{m_\rho^2 - m_\omega^2 + i\,m_\omega\Gamma_\omega} \right| = 5.8\,,$$

which transforms the intervals (27), (28) into

$$0.14 \leq r_{\rho 0/\rho\pm\pi} \leq 2.65\,,$$
$$0.56 \cdot 10^{-4} \leq \mathrm{Br}\,(\rho^0 \to \pi^0\gamma) \leq 13.25 \cdot 10^{-4}\,.$$

The upper ends here just agree with the estimates of ref. [3]. However, derivation in the previous section demonstrates that the mixing parameter for production and decay amplitudes (see expressions (14), (15)) should contain in its denominator the difference of pole masses, even though k^2 in propagators takes only real values and does not reach any of the pole (complex) masses.

Let us discuss the experimental situation. The latest version of Particle Data Tables [1] gives the value

$$\mathrm{Br}\,(\rho^0 \to \pi^0\gamma) = (7.9 \pm 2.0) \cdot 10^{-4}\,,$$

based on one experiment [12] only. Reanalysis of all existing data on $e^+e^- \to \pi^0\gamma$ was presented in [13] with taking into account coherent contributions of various resonances. It provided, with some model assumptions, two sets of acceptable solutions for $\mathrm{Br}\,(\rho^0 \to \pi^0\gamma)$, one between $6 \cdot 10^{-4}$ and $7 \cdot 10^{-4}$, another between $11 \cdot 10^{-4}$ and $12 \cdot 10^{-4}$, all higher than $\mathrm{Br}\,(\rho^\pm \to \pi^\pm\gamma) = (4.5 \pm 0.5) \cdot 10^{-4}$ [1]. The previous version of Particle Data Tables [6] used the lower solution for a particular model, though ref. [13] gave only meager motivations for this model and this solution. There are arguments showing that the results [13] for $\rho^0 \to \pi^0\gamma$ are still rather uncertain: the models used assume non-PDG values of m_ρ and/or Γ_ρ; the triangle anomaly contribution is assumed to be the only nonresonant background, but the out-of-resonance data cannot confirm its presence in $e^+e^- \to \pi^0\gamma$ (though do confirm the similar anomaly contribution to $e^+e^- \to \eta\gamma$); phase relations between various resonance contributions look strange and unexpected. The own conclusion of the authors of ref. [13] is that more measurements, with better accuracy, are necessary for the $\pi^0\gamma$ final state to obtain a firm result.

Meanwhile, the above value used in the latest tables [1] looks acceptable at the moment, just due to its large error. Though with such or even larger uncertainties, all published measurements give evidence for the enhancement of $\rho^0 \to \pi^0\gamma$ in respect to $\rho^\pm \to \pi^\pm\gamma$ and, thus, evidence for the negative sign of $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$.

3.3 Decays $(\omega, \rho) \to \eta\gamma$

Assumptions, which lead to relations (23), provide similar relations also for amplitudes of some other decays. *E.g.*, for decays to $\eta\gamma$ we obtain

$$3\,A^{(0)}(\omega \to \eta\gamma) = A^{(0)}(\rho^0 \to \eta\gamma)\,. \qquad (31)$$

The factor 3 has here the same nature as in eq. (23), though it makes more intensive (surely, in terms of partial widths, not of branchings) decay of ρ^0 instead of ω.

For the final state $\eta\gamma$ we have no analog of the modes $\rho^\pm \to \pi^\pm\gamma$, insensitive to the $(\rho\omega)$-mixing. Nevertheless, in analogy with the ratio $r_{\omega/\rho 0\pi}$ of eq. (29), we can construct another ratio

$$r_{\rho 0/\omega\eta} \equiv \frac{\Gamma(\rho^0 \to \eta\gamma)}{\Gamma(\omega \to \eta\gamma)} = 9 \left| \frac{1 - \frac{1}{6}\,\widetilde{G}_{\rho\omega}}{1 + \frac{3}{2}\,\widetilde{G}_{\omega\rho}} \right|^2 . \qquad (32)$$

With 2σ boundaries for $|\widetilde{G}|$ it has the admissible interval

$$7.2 \leq r_{\rho 0/\omega\eta} \leq 11.6\,, \qquad (33)$$

numerically the same as in eq. (30). Note, however, different correlations: the lower/upper boundary in eq. (33) corresponds to the lower/upper boundaries in eqs. (27), (28), but to the upper/lower boundary in eq. (30). In terms of $\mathrm{Re}\,\widetilde{G}$ the lower/upper boundary in eq. (33) corresponds to positive/negative $\mathrm{Re}\,\widetilde{G}$, opposite to eq. (30).

Let us consider the present experimental situation. Particle Data Group [1] gives, after all evaluations,

$$\mathrm{Br}\,(\rho^0 \to \eta\gamma) = (3.8 \pm 0.7) \cdot 10^{-4}\,,$$
$$\Gamma(\rho^0 \to \eta\gamma) = (57 \pm 10)\ \mathrm{keV}\,, \qquad (34)$$

and

$$\mathrm{Br}\,(\omega \to \eta\gamma) = (6.5 \pm 1.1) \cdot 10^{-4}\,,$$
$$\Gamma(\omega \to \eta\gamma) = (5.5 \pm 0.9)\ \mathrm{keV}\,. \qquad (35)$$

This implies

$$r_{\rho 0/\omega\eta} = 10.3 \pm 2.6\,, \tag{36}$$

in agreement with the interval (33). This value may be considered as an additional evidence for $\mathrm{Re}\,\widetilde{G}_{\omega\rho} < 0$ and, therefore, as an indirect evidence for enhancement of $\rho^0 \to \pi^0\gamma$ due to $(\rho\omega)$-mixing. However, the large error of the value (36) makes this result rather uncertain.

3.4 Decays $\eta' \to (\omega, \rho)\,\gamma$

Bare amplitudes of these decays are related just as in decays with η-meson:

$$3\,A^{(0)}(\eta' \to \omega\gamma) = A^{(0)}(\eta' \to \rho^0\gamma)\,. \tag{37}$$

Therefore, similar to $r_{\rho 0/\omega\eta}$, we can construct the ratio

$$r_{\eta'\rho 0/\omega} \equiv \frac{\Gamma(\eta' \to \rho^0\gamma)}{\Gamma(\eta' \to \omega\gamma)} = 9\left|\frac{1 - \frac{1}{6}\,\widetilde{G}_{\omega\rho}}{1 + \frac{3}{2}\,\widetilde{G}_{\rho\omega}}\right|^2 \tag{38}$$

with the same boundaries

$$7.2 \le r_{\eta'\rho 0/\omega} \le 11.6\,. \tag{39}$$

Its correlations with other similar ratios are also the same as for $r_{\rho 0/\omega\eta}$.

Experimental data [1] give

$$\begin{aligned} \mathrm{Br}\,(\eta' \to \rho^0\gamma) &= (29.5 \pm 1.0)\%\,,\\ \mathrm{Br}\,(\eta' \to \omega\gamma) &= (3.03 \pm 0.31)\%\,, \end{aligned} \tag{40}$$

that lead to the value

$$r_{\eta'\rho 0/\omega} = (9.74 \pm 1.05) \tag{41}$$

inside the expected interval (39). It looks to be shifted upward from 9, thus giving evidence for $\mathrm{Re}\,\widetilde{G}_{\rho\omega} < 0$ and the enhanced decay $\rho^0 \to \pi^0\gamma$. But such small shift with rather large error still prevents one from any firm conclusion.

3.5 Decays $(\omega, \rho) \to e^+e^-$

Decay of a neutral C-odd vector meson to e^+e^- pair goes through one virtual photon. If the meson may be considered to consist of a quark-antiquark pair, the decay amplitude should be equal to the quark charge e_q multiplied by some hadronic matrix element. (In terms of the constituent-quark picture it is proportional to the short-distance value of the internal wave function.)

The situation is somewhat different for ω and ρ. Here, even for bare states, $\omega^{(0)}$ and $\rho^{(0)}$, we have coherent mixtures of at least two flavours with different charges: $(\overline{u}u + \overline{d}d)/\sqrt{2}$ and $(\overline{u}u - \overline{d}d)/\sqrt{2}$. Here we can use some averaged charges as the effective charges e_ω and e_ρ.

If direct isospin violation is absent (or may be neglected), so that the arising hadronic matrix elements are the same for $\overline{u}u$ and $\overline{d}d$ components, then annihilation of the bare states (transforming them into vacuum) by the quark e.m. current provides the effective charges in the form

$$e_\omega = \frac{e_u + e_d}{\sqrt{2}} = \frac{1}{3\sqrt{2}}\,, \qquad e_\rho = \frac{e_u - e_d}{\sqrt{2}} = \frac{1}{\sqrt{2}}\,.$$

Note the relative factor 3 that appears here again. It is natural, therefore, to expect that bare decay amplitudes satisfy relations similar to eqs. (31), (37):

$$3\,A^{(0)}(\omega^{(0)} \to e^+e^-) = A^{(0)}(\rho^{(0)} \to e^+e^-)\,. \tag{42}$$

Thus, in full similarity to previous cases, one can construct the ratio for physical quantities

$$r_{\rho 0/\omega(ee)} \equiv \frac{\Gamma(\rho^{(0)} \to e^+e^-)}{\Gamma(\omega^{(0)} \to e^+e^-)} = 9\left|\frac{1 - \frac{1}{6}\,\widetilde{G}_{\rho\omega}}{1 + \frac{3}{2}\,\widetilde{G}_{\omega\rho}}\right|^2\,, \tag{43}$$

again with the same boundaries

$$7.2 \le r_{\rho 0/\omega(ee)} \le 11.6\,, \tag{44}$$

and the same correlations with other similar ratios and with the sign of $\mathrm{Re}\,\widetilde{G}$ as for $r_{\rho 0/\omega\eta}$ or $r_{\eta'\rho 0/\omega}$. According to the present experimental data [1]

$$\begin{aligned} \Gamma(\rho^0 \to e+e-) &= (6.85 \pm 0.11)\mathrm{keV}\,,\\ \Gamma(\omega \to e+e-) &= (0.60 \pm 0.02)\mathrm{keV}\,,\\ r_{\rho 0/\omega(ee)} &= (11.42 \pm 0.42)\,. \end{aligned} \tag{45}$$

This value reasonably agrees with the values (36), (41) and gives a clearer evidence for $\mathrm{Re}\,\widetilde{G}_{\omega\rho} < 0$.

Decays to e^+e^- seem to admit even a more detailed test of the mixing picture. For heavy quarkonia, where hypothesis of one quark-antiquark pair looks fulfilled, there is an empirical observation that the partial widths of their decays to e^+e^- equals just a constant multiplied by the corresponding quark charge squared:

$$\Gamma(Q\overline{Q} \to e^+e^-) = e_Q^2\,\Gamma_0\,. \tag{46}$$

Indeed, let us consider three heavier quarkonia, Υ, J/ψ, φ corresponding (with good accuracy) to the definite flavour of the constituent quark (and antiquark) and, hence, to the definite value of $e_Q : e_b = -1/3,\ e_c = 2/3,\ e_s = -1/3$. Then, from experimental data [1],

$$\begin{aligned} \Gamma_0^{(\Upsilon)} &= (11.88 \pm 0.45)\ \mathrm{keV}\,,\\ \Gamma_0^{(J/\psi)} &= (11.84 \pm 0.83)\ \mathrm{keV}\,,\\ \Gamma_0^{(\varphi)} &= (11.34 \pm 0.18)\ \mathrm{keV}\,. \end{aligned} \tag{47}$$

We can try to check this regularity for ρ^0, ω as well, using effective charges e_ρ, e_ω. Then the data [1] lead to values

$$\begin{aligned} e_\omega^{-2}\,\Gamma(\omega \to e^+e^-) &= 10.80 \pm 0.36\ \mathrm{keV}\,,\\ e_\rho^{-2}\,\Gamma(\rho^0 \to e^+e^-) &= 13.70 \pm 0.22\ \mathrm{keV}\,, \end{aligned} \tag{48}$$

which look somewhat lower (for ω) or higher (for ρ) than the "normal" values (47).

There are at least three possible explanations: 1) insufficient precision prevents from any statement of differences between numerical values (47) and (48); 2) the present level of understanding QCD is not sufficient for extrapolating the properties of heavy quarkonia to lighter ones; 3) values (48) for physical mesons ρ, ω may deviate from (47) due to mixing of bare states $\rho^{(0)}$, $\omega^{(0)}$. The first two points imply that any serious discussion should be postponed till further experimental and/or theoretical progress. Therefore, we will not touch them here and now; instead we restrict ourselves to the third possibility.

Let us assume that the above regularity would be correct for the bare states $\rho^{(0)}$, $\omega^{(0)}$ and that Γ_0 is indeed a universal quantity (without discussing why). Then the $(\rho\omega)$-mixing changes widths for the physical states ρ^0, ω so that

$$e_\omega^{-2}\,\Gamma(\omega \to e^+e^-) = \Gamma_0 \left|1 + \frac{3}{2}\,\widetilde{G}_{\omega\rho}\right|^2 ,$$
$$e_\rho^{-2}\,\Gamma(\rho^0 \to e^+e^-) = \Gamma_0 \left|1 - \frac{1}{6}\,\widetilde{G}_{\rho\omega}\right|^2 . \qquad (49)$$

In such a framework the relation of numerical values (48), for ρ^0 higher than for ω, gives a new strong support to $\mathrm{Re}\,\widetilde{G}_{\rho\omega} < 0$ and, hence, to the enhancement of the mode $\rho^0 \to \pi^0\gamma$ in respect to $\rho^\pm \to \pi^\pm\gamma$.

Further, taking for definiteness the heavier quarkonium value $\Gamma_0 = 11.86$ keV from eq. (47), we obtain the expected intervals

$$9.44 \text{ keV} \le e_\omega^{-2}\,\Gamma(\omega \to e^+e^-) \le 14.56 \text{ keV}\,, \qquad (50)$$

and

$$11.58 \text{ keV} \le e_\rho^{-2}\,\Gamma(\rho^0 \to e^+e^-) \le 12.15 \text{ keV}\,. \qquad (51)$$

Quantitatively, the value (48) for ω is in good agreement with the interval (50), while the value for ρ^0 noticeably exceeds the expected upper boundary (51). This could mean either that $\Gamma_0^{(\rho,\omega)}$ deviates from $\Gamma_0^{(\Upsilon)} \approx \Gamma_0^{(J/\psi)}$ or even that $\Gamma_0^{(\rho)} \neq \Gamma_0^{(\omega)}$ due, *e.g.*, to direct isospin violation for decay amplitudes of the bare states (see discussion below). Having in mind the universal character of the mixing parameter $\widetilde{G}_{\rho\omega} = \widetilde{G}_{\omega\rho}$, one may hope that precise investigation of a wider set of processes will allow to clarify the situation.

One may add here one more notice. Of course, all three values (47) coincide at the level not worse than 1σ. Nevertheless, $\Gamma_0^{(\Upsilon)}$ and $\Gamma_0^{(J/\psi)}$ are equal to each other with much better accuracy, while $\Gamma_0^{(\varphi)}$ is somewhat lower. Such situation, if confirmed, could be due to mixing of φ with ω and/or other mesons.

4 Discussion

As was demonstrated in the preceding section, the $(\rho\omega)$-mixing manifests itself not only in the well-known decay $\omega \to \pi^+\pi^-$ and in radiative decay $\rho^0 \to \pi^0\gamma$, but also in some other electromagnetic decays with participation of ρ or ω, in either initial or final state. Particular modes of interest are radiative decays $(\rho^0,\omega) \to \eta\gamma$, $\eta' \to (\rho^0,\omega)\gamma$ and decays $(\rho^0,\omega) \to e^+e^-$ going through one virtual photon. The central point of studies appears to be a special consistent correlation between properties of decays in various pairs.

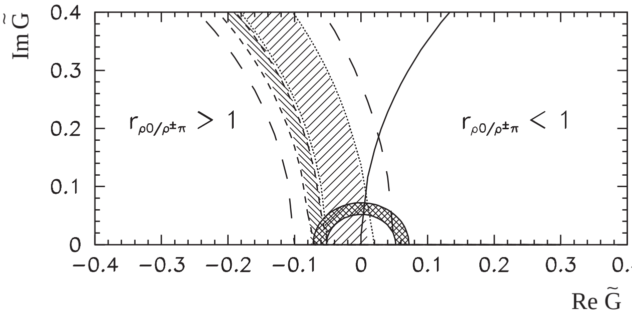

Fig. 1. Data on various decay pairs as seen at the complex plane of $\widetilde{G}$. The long-dashed uncovered band is for $r_{\rho0/\omega\eta}$, eq. (36); the short-dashed band with left-inclined hatching is for $r_{\rho0/\omega(ee)}$, eq. (45); the dotted band with right-inclined hatching is for $r_{\eta'\rho0/\omega}$, eq. (41). The solid ring with double hatching is for $(\omega,\rho) \to \pi\pi$, eq. (21) with 2σ width. Space to the left/right of the solid line corresponds to $r_{\rho0/\rho\pm\pi}$ more/less than unity, *i.e.*, to enhancement/suppression of $\rho^0 \to \pi^0\gamma$ in respect to $\rho^\pm \to \pi^\pm\gamma$.

The existing data for the decay pairs may be presented on the complex plane of the mixing parameter $\widetilde{G}$, as seen at fig. 1. If the role of mixing for the decays is correctly described in the preceding section, then all the corresponding bands should overlap. The presently achieved accuracy is not yet sufficiently informative. However, the data do not contradict to overlapping at $\mathrm{Re}\,\widetilde{G} < 0$, which corresponds to an enhancement of $\rho^0 \to \pi^0\gamma$ in respect to $\rho^\pm \to \pi^\pm\gamma$. Being done with better precision, experiments on those (and other) decays could check the expected correlation of properties of different processes and, thus, confirm (or reject) the role of mixing.

Let us analyze the nature of that correlation. The first step in its description begins with relations for bare (unmixed) amplitudes. At first sight, the used relations (23), (31), (37), (42) may be justified only in the framework of a specific (constituent quark) model. However, they have a more general origin. Indeed, electromagnetic interactions of hadrons in the quark picture are manifestations of the "microscopic" interaction proportional (for light quarks) to

$$e_u\,\overline{u}u + e_d\,\overline{d}d = \frac{e_u + e_d}{\sqrt{2}}\,\frac{\overline{u}u + \overline{d}d}{\sqrt{2}} + \frac{e_u - e_d}{\sqrt{2}}\,\frac{\overline{u}u - \overline{d}d}{\sqrt{2}}$$

(of course, we mean the vector current structure, well-known and not shown here explicitly in detail). Hence, the canonical quark charges imply that the isovector component of the photon is coupled to light quarks 3 times stronger than the isoscalar one.

All the considered pairs of decays, differing by interchange $\rho^0 \rightleftharpoons \omega$, have a common property: one of them contains only the isovector component of the photon (real or virtual), while only the isoscalar component of the photon participates in another decay. Relations (23), (31), (37), (42) correspond to the simple expectation from the above discussion that amplitudes for light-quark processes with the isovector photon are 3 times larger than that for similar processes with isoscalar photon. Of course, these simple relations may be modified in particular processes by specific hadronic matrix elements. Nevertheless, one may argue that the modifications should not be large.

Indeed, the processes discussed here are soft, and essential contributions to their amplitudes come from the photon coupling to valence quarks. Now, since the valence quark structure inside any hadron is similar to the constituent quark one, it is natural to expect that relations for bare amplitudes are closely similar to the used relations (23), (31), (37), (42). Such arguments seem to be applicable both for radiative decays and for e^+e^--decays of mesons. Note that similar reasoning might also explain why QCD calculations (say, the sum rules) and constituent quarks provide nearly the same predictions for meson radiative decays.

The above relations between processes with isovector *vs.* isoscalar photon might be applicable to amplitudes for "bare" (unmixed) states, where isospin could be a good quantum number. Then the next step should be the description of isospin violation by the $(\rho\omega)$-mixing. It makes the relations for physical (mixed) amplitudes of the decays to be somewhat modified in comparison with those for bare amplitudes. An essential point is that different decay pairs are modified in a correlated way, since in all cases the mixing is described by the same universal dimensionless (generally, complex) phenomenological parameter $\widetilde{G}_{\rho\omega}\ (= \widetilde{G}_{\omega\rho})$.

Future accurate experiments should allow to check whether all those correlations are correct and, thus, examine the consistency of the picture. But some piece of information does exist even now. Data on $\omega \to \pi^+\pi^-$ allow to find the absolute value of the mixing parameter $|\widetilde{G}_{\rho\omega}|$. If the decays $(\rho^0, \omega) \to \pi^0\gamma$ were measured at least with the same precision as $\rho^\pm \to \pi^\pm\gamma$, we could extract also $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$ and then test hypotheses on the mechanism of the $(\rho\omega)$-mixing.

Meanwhile, the existing direct measurements give evidence for enhancement of $\rho^0 \to \pi^0\gamma$ in comparison with $\rho^\pm \to \pi^\pm\gamma$ (the exact number is still to be determined). This implies that $\mathrm{Re}\,\widetilde{G}_{\rho\omega} < 0$ and suggests a special kind for modification of amplitudes in other pairs of decays with participation of ρ^0, ω.

As was demonstrated in the preceding section and in fig. 1, current data on $(\rho^0, \omega) \to \eta\gamma$, $\eta' \to (\rho^0, \omega)\gamma$ and $(\rho^0, \omega) \to e^+e^-$ are not confirmative yet, but nevertheless give additional support for negative $\mathrm{Re}\,\widetilde{G}_{\rho\omega}$ and, therefore, indirectly confirm the enhancement of $\rho^0 \to \pi^0\gamma$. Since $\widetilde{G}_{\rho\omega} = G_{\rho\omega}/\delta M^2$ with nearly imaginary $\delta M^2 = (M_\omega^2 - M_\rho^2)/2$, this implies also that the direct $(\rho\omega)$ vertex $G_{\rho\omega}$ is not real and, as a result, may reject even now some simplified models for the $(\rho\omega)$-transition.

The general character of the used relation between isovector and isoscalar components of the photon may be checked by testing it in a wider set of decays after they become accessible. As an example we can take the pair of decays $(\rho^0, \omega) \to \pi^0\pi^0\gamma$, where the photon has $I = 1$ for ρ^0-decay and $I = 0$ for ω-decay. Particle Data Tables [1] give

$$\begin{aligned} \mathrm{Br}\,(\rho^0 \to \pi^0\pi^0\gamma) &= (4.8^{+3.4}_{-1.9}) \cdot 10^{-5}\,, \\ \mathrm{Br}\,(\omega \to \pi^0\pi^0\gamma) &= (7.8 \pm 3.4) \cdot 10^{-5}\,. \end{aligned} \tag{52}$$

Together with data on total widths we obtain

$$r_{\rho 0/\omega(\pi 0\pi 0)} \equiv \frac{\Gamma(\rho^0 \to \pi^0\pi^0\gamma)}{\Gamma(\omega \to \pi^0\pi^0\gamma)} \approx 11\,. \tag{53}$$

The large experimental uncertainty of branchings (52) prevents us from a more detailed discussion of these decays. However, they may be useful and helpful in future studies. But even at present one can notice close equality of $r_{\rho 0/\omega(\pi 0\pi 0)}$ to other r-ratios of the previous section. This confirms the universality of stronger isovector *vs.* isoscalar interaction for the photon, just at the expected quantitative level.

All numerical estimations in this paper have been made under the assumption that all necessary parameters are constant. Those parameters are ρ, ω complex masses (*i.e.*, masses and widths) and mixing parameters G (or $\widetilde{G}$). Such approach is analogous to the standard Breit-Wigner description of a resonance amplitude, with fixed values of energy (mass) and width. It is known to work quite well for the description of narrow peaks, as ω or φ. However, to describe the broad ρ peak one needs to account for the k^2-dependence of, at least, the ρ width. Moreover, even to describe the $(\varphi\omega)$-interference in $e^+e^- \to \pi^+\pi^-\pi^0$, one seems to need the "long tail" of the ω-resonance, with taking account of the k^2-dependence of its width [14]. These examples show that, most probably, detailed description of, say, $e^+e^- \to \pi^0\gamma$ for the extraction of the partial ρ width and $(\rho\omega)$-interference may require to consider the k^2-dependence of the parameters (at least, at future level of precision).

Another simplifying hypothesis used was the leading role of the $(\rho\omega)$-mixing for isospin violation. A simple structure was assumed for bare amplitudes, corresponding to a "minimal" violation of isospin[3]. There are, however, arguments in favour of the necessity of a more structure, with direct violation of the isospin in bare amplitudes.

Indeed, let us consider first the radiative decays. In terms of the constituent quarks their transition amplitudes are determined by the magnetic moments of the quarks, which manifest themselves also in baryon magnetic moments. The factor 3 in relations (23), (31), (37)

[3] Since all the discussed decays, except may be $(\rho, \omega) \to \pi^+\pi^-$, are evidently electromagnetic, they surely violate isospin. But the violation accounted for was only due to difference of the charges e_u, e_d, without taking account of the difference of quark masses or other properties.

corresponds to the assumption that the magnetic moments of u, d are equal to their charges e_u, e_d multiplied by the same factor. Since this assumption implies also the ratio of the proton/neutron magnetic moments $\mu_p/\mu_n = -3/2$, we know that it is only approximate. The well-established (small) deviation of this magnetic-moments ratio from $-3/2$ gives evidence for the difference of factors in the quark magnetic moments (the same conclusion comes from the magnetic moments of other baryons), due to different masses or because of other reasons[4]. Thus, the factor 3 should be corrected, and the corrections can be extracted from the existing data[5]. However, repeating the analysis of section 3 with these corrections shows that today they appear to be corrections to corrections in comparison with the effects of the $(\rho\omega)$-mixing.

Arguments for direct isospin violation in e^+e^--decays look different. It is essential here that both $\rho^{(0)}$ and $\omega^{(0)}$ have two flavour components, their couplings to photon being proportional just to the charges e_u, e_d. Thus, each of the bare decay amplitudes is a combination of two flavour contributions. According to the constituent-quark model, every contribution due to annihilation of a pair $\overline{Q}Q$ is proportional to the product of e_Q and the corresponding short-range wave function $\psi_Q(0)$. Exact isospin conservation implies equality $\psi_u(0) = \psi_d(0)$. However, the scaling relation (46) leads to the phenomenological dependence on the quark mass [15]

$$|\psi_Q(0)|^2 \propto m_Q^2$$

(note that it would be m_Q^3 for the Coulomb-like potential). Now, the inequality of m_u and m_d should influence the amplitudes $\rho^{(0)} \to e^+e^-$, $\omega^{(0)} \to e^+e^-$ and deviate their ratio from 3 (recall that we deal here with constituent quarks, so the correction should be at a level of several percents, not several times as it would be for current quark masses)[6].

Of course, there are also some other corrections. *E.g.*, ω contains an admixture of strange quarks which may be described as a mixing of ω and φ with mixing angle $\alpha_V \approx 4°$. Corresponding relative corrections for decays of section 3 are of order $\sim \sin^2 \alpha_V \approx 5{\cdot}10^{-3}$. Their influence appears even smaller than discussed above and may be necessary only at future levels of precision.

[4] It is interesting to note that the corresponding factor for the heavier d-quark appears 5% larger than for lighter u-quark, contrary to familiar properties of normal magnetic moments. This can be viewed as evidence that (constituent) quarks may have anomalous magnetic moments.

[5] In the framework of the constituent-quark model it is 3.21 instead of 3 in eqs. (23), (31), (37), which gives the factor 10.3 instead of 9 for r's, in agreement with the present experimental value (25).

[6] Interestingly enough, this mechanism acts in the same direction as mixing: it enhances $\rho^{(0)} \to e^+e^-$ and suppresses $\omega^{(0)} \to e^+e^-$, thus increasing the coefficient in eq. (42). Such changes are favourable, since they shift the theoretically expected intervals (50), (51) just so to adjust them to experimental values (48).

5 Conclusions

Results of the present paper may be briefly summarized as follows.

1. Independently of a framework of any effective field theory, the $(\rho\omega)$-mixing is completely determined by two universal parameters $\widetilde{G}_{\rho\omega}$ and $\widetilde{G}_{\omega\rho}$, both being, generally, complex. For the case of T conservation (or in the framework of effective field theory) they may be made equal to each other, staying complex outside the effective field theory. Experimental determination of the mixing parameter(s) would allow to check models of isospin violation.
2. It was known for many years that isospin violation, due to the $(\rho\omega)$-mixing, should be enhanced in the forbidden decay $\omega \to \pi^+\pi^-$; later the same effect was suggested for the radiative decay $\rho^0 \to \pi^0\gamma$ (its branching ratio may be unequal to that of $\rho^\pm \to \pi^\pm\gamma$ due to the interference of the direct decay and cascade decay $\rho^0 \to \omega \to \pi^0\gamma$). As shown here, the mixing should also affect all pairs of decays of ρ^0, ω to the same final state and decays of heavier particles with production of ρ^0, ω.
3. The $(\rho\omega)$-mixing influences various pairs of (ρ^0, ω)-decays in a regular, correlated manner. Such regularities agree with the existing data on radiative decays and decays to e^+e^-, though the achieved precision of data is insufficient for a firm conclusion. Nevertheless, the data prefer $\mathrm{Re}\,\widetilde{G}_{\rho\omega} < 0$. This, on the one side, supports the enhancement of $\rho^0 \to \pi^0\gamma$ in comparison with $\rho^\pm \to \pi^\pm\gamma$ and implies, on the other, that the direct $(\rho\omega)$-coupling $G_{\rho\omega} = G_{\omega\rho}$ is complex.
4. The universal nature of the mixing parameter will allow, at higher experimental accuracy, to separate the mixing isospin violation due to $(\rho\omega)$ transitions from direct isospin violation in amplitudes of the "bare" (unmixed) states $\rho^{(0)}$ and $\omega^{(0)}$. Even the present data give some evidence of the necessity of such direct violating effects.

Thus, we can expect that in the near future the meson radiative decays with participation of ρ and/or ω may indeed be attractive and useful for studying the $(\rho\omega)$-mixing and other manifestations of isospin violation.

About forty years ago, in the first years of the quark era, the radiative decays of mesons were suggested (and really used) as a mean to check that quarks inside baryons and mesons are the same [7–10,16]. Now, forty years later, at a higher level of experimental precision and theoretical understanding, such decays may again provide new interesting information. This time the radiative decays might elucidate mechanisms of isospin violation.

I thank V.V. Anisovich for stimulating discussions and V.A. Nikonov for help in preparing the text. The work was supported in part by the RFBR grant 00-15-96610.

References

1. K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
2. M. Benayoun, H.B.O'Connell, Eur. Phys. J. C **22**, 503 (2001); nucl-th/0107047.
3. X.J. Wang, J.H. Jiang, M.L. Yan, Eur. Phys. J. A **14**, 219 (2002); hep-ph/0101166.
4. T.D. Lee, R. Oehme, C.N. Yang, Phys. Rev. **106**, 340 (1957).
5. X.J. Wang, M.L. Yan, Phys. Rev. D **62**, 094013 (2000); hep-ph/0003218.
6. D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
7. Ya.I. Azimov, V.V. Anisovich, A.A. Anselm *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **1**, 50 (1965); JETP Lett. **1**, 72 (1965).
8. V.V. Anisovich, A.A. Anselm, Ya.I. Azimov *et al.*, Phys. Lett. **16**, 194 (1965).
9. L.D. Soloviev, Phys. Lett. **16**, 345 (1965).
10. C. Becchi, G. Morpurgo, Phys. Rev. B **140**, 687 (1965).
11. R. Shrock, hep-th/0204005.
12. S.I. Dolinsky *et al.*, Z. Phys. C **42**, 511 (1989).
13. M. Benayoun, S.I. Eidelman, V.N. Ivanchenko, Z. Phys. C **72**, 221 (1996).
14. N.N. Achasov, A.A. Kozhevnikov, Phys. Rev. D **61**, 054005 (2000); hep-ph/9906520.
15. Ya.I. Azimov, L.L. Frankfurt, V.A. Khoze, Usp. Fiz. Nauk **124**, 459 (1978); Sov. Phys. Usp. **21**, 225 (1978).
16. W.E. Thirring, Phys. Lett. **16**, 335 (1965).